Journal of Computational and Applied Mathematics 230 (2009) 443–450

Contents lists available at ScienceDirect

Journal of Computational and Applied Mathematics

journal homepage: www.elsevier.com/locate/cam

A one-model approach based on relaxed combinations of inputs for evaluating input congestion in DEA

Mohammad Khodabakhshi *

Department of Mathematics, Faculty of Science, Lorestan University, Khorram Abad, Iran

ARTICLE INFO

Article history:
Received 17 May 2008
Received in revised form 3 October 2008

Keywords:
Data envelopment analysis
Congestion
Inefficiency
Relaxed inputs

ABSTRACT

This paper provides a one-model approach of input congestion based on input relaxation model developed in data envelopment analysis (e.g. [G.R. Jahanshahloo, M. Khodabakhshi, Suitable combination of inputs for improving outputs in DEA with determining input congestion — Considering textile industry of China, Applied Mathematics and Computation (1) (2004) 263–273; G.R. Jahanshahloo, M. Khodabakhshi, Determining assurance interval for non-Archimedean ele improving outputs model in DEA, Applied Mathematics and Computation 151 (2) (2004) 501–506; M. Khodabakhshi, A super-efficiency model based on improved outputs in data envelopment analysis, Applied Mathematics and Computation 184 (2) (2007) 695–703; M. Khodabakhshi, M. Asgharian, An input relaxation measure of efficiency in stochastic data analysis, Applied Mathematical Modelling 33 (2009) 2010–2023]. This approach reduces solving three problems with the two-model approach introduced in the first of the above-mentioned reference to two problems which is certainly important from computational point of view. The model is applied to a set of data extracted from ISI database to estimate input congestion of 12 Canadian business schools.

1. Introduction

Data Envelopment Analysis (DEA) was originated in 1978 by Charnes, Cooper and Rhodes (CCR) [1]. Later, Banker, Charnes and Cooper (BCC) [2] introduced a variable returns to scale version of the CCR model, the so-called BCC model in 1984. Since 1978 there has been a surge of research on DEA, and many authors all over the world are working in this area. The objective of DEA models is evaluating performances of decision making units (DMUs). The performances of DMUs are affected by the amount of sources that DMUs used. For example, congesting sources will interfere to produce more outputs than the observed output of DMU being evaluated. It is important to identify DMUs that have input congestion as well as estimate the value of their input congestion. Contemporary research into the congestion topic long neglected in the economics was begun in [3]. The concepts in this article were subsequently given operationally implementable form first in [4] and then in [5] in the form of models (and methods of analysis) that would now be identified with data envelopment analysis (DEA). However, in both economics and OR literature the pace of research into congestion quickened after in [5] publication. Cooper et al. [6], also, published an alternative approach for determining input congestion that have some advantages to previous method. To see discussions on alternative approaches of congestion refer to [7–10]. See [11] which developed a unified additive model for determining congestion, too. In this paper, we will develop a one-model approach to estimate input congestion of the evaluating DMU by output improvement model (see [12–14]). Although the proposed method is similar to that of

* Tel.: +98 9122586521.
E-mail address: mkhbakhshi@yahoo.com.

0377-0427/$ – see front matter
doi:10.1016/j.cam.2008.12.028

Table 1
Numerical results of the model (1).

DMU	(I, O)	ϕ_o^*	s_{i1}^{-*}	s_{i2}^{+*}	s_r^{+*}	Result
A	(1, 0.5)	4	0	1	0	Inefficient
B	(2, 2)	1	0	0	0	Efficient
C	(3, 2)	1	1	0	0	Efficient
D	(5, 1)	2	3	0	0	Inefficient

Cooper et al. [15], it is based on relaxed combinations of inputs, see Jahanshahloo and Khodabakhshi [12]. It is shown that it detects and determines congestion and its amount. Furthermore, it reduces computation time in practical applications.

The paper is organized as follows: In the next section, we will introduce input relaxation model. In Section 3, we discuss congestion; a one-model approach for determining input congestion will also be provided. An empirical example will be studied in Section 4. Section 5 concludes the paper.

2. The input relaxation model

Jahanshahloo and Khodabakhshi [12,13] introduced the following model for improving output:

$$\begin{aligned}
\text{Maximize}\quad & \phi_o + \varepsilon\left(\sum_{i=1}^{m} s_{i1}^- + \sum_{r=1}^{s} s_r^+ - \sum_{i=1}^{m} s_{i2}^+\right)\\
\text{Subject to}\quad & x_{io} = \sum_{j=1}^{n}\lambda_j x_{ij} + s_{i1}^- - s_{i2}^+, \quad i = 1,\ldots,m\\
& 0 = \sum_{j=1}^{n}\lambda_j y_{rj} - \phi_o y_{ro} - s_r^+, \quad r = 1,\ldots,s\\
& 1 = \sum_{j=1}^{n}\lambda_j\\
& s_{i1}^-, s_{i2}^+, \lambda_j, s_r^+ \geq 0.
\end{aligned} \tag{1}$$

Since $s_{i1}^- = 0, s_{i2}^+ = 0\ (\forall i), \lambda_j = 0\ (j \neq 0), \lambda_0 = 1, \forall r\ s_r^+ = 0, \phi_0 = 1$ is a feasible solution, the model is always feasible. This model is a version of output oriented BCC model [2] which is not limited in using observed sources of evaluating DMU. While output oriented BCC model is limited in using observed sources of evaluating DMU, input relaxation model – by imposing a few changes on some inputs – can produce outputs more than observed output or output suggested by BCC model. Using the above model will be very useful when attracting some inputs such as labor which is necessary to solve employment problem in a society, see Jahanshahloo and Khodabakhshi [12] where the model was used in textile industry of China. The conditions of efficiency under the above model for DMU_o being evaluated become as follows:

Definition 1 (*Efficiency*). DMU_o is efficient under model (1) if the following two conditions are satisfied:

(i) $\phi_o^* = 1$
(ii) Optimal amounts of all slacks are zero.

See also [14] to rank efficient units.

Example. Consider Table 1 in which columns 1 and 2 provide four DMUs with associated input–output data. The first number in the parenthesis of the column 2 indicates an input amount and the second one an output amount for DMUs labeled A to D. The numerical results of model (1) are presented in Table 1. This model for DMU_A becomes:

$$\begin{aligned}
\text{Maximize}\quad & \phi_A + \varepsilon(s_{11}^- - s_{12}^+ + s^+)\\
\text{Subject to}\quad & 1 = \lambda_A + 2\lambda_B + 3\lambda_C + 5\lambda_D + s_{11}^- - s_{12}^+\\
& 0 = 0.5\lambda_A + 2\lambda_B + 2\lambda_C + \lambda_D - 0.5\phi_A - s^+\\
& 1 = \lambda_A + \lambda_B + \lambda_C + \lambda_D\\
& s_{11}^-, s_{12}^+, s^+\lambda_A, \lambda_B, \lambda_C, \lambda_D \geq 0.
\end{aligned}$$

Optimal solution in evaluating DMU A is $\phi_A^* = 4, s_{12}^{+*} = 1, \lambda_B^* = 1$, and other variables are zero that means if the input of DMU A becomes twice, its output will be fourfold. In fact, at first its input is increased by $s_{12}^{+*} = 1$, and then for new input the previous output is obtained by four times.

Solving input relaxation model is a two-stage method. At the first stage, we obtain max ϕ_o subject to constraints of (1), and at the second stage by using optimal value of ϕ_o^* we proceed to optimize max slacks with fixing ϕ_o^* instead of ϕ_o. Therefore, in this method we do not assign any amount to ε for solving the model.

Hereunder, we describe a two-model approach for determining input congestion (see [12]) by output improvement model, and then we proceed to develop a one-model approach.

3. Congestion

If some inputs are used in amounts, they cause output reduction, then input congestion exists. For instance, an excess of miners bumping into each other in an underground mine is an example, where a reduction in the number of miners can result in an increase in the amount mined. In what follows, the exact definition of congestion in general case is provided.

Definition 2 (*Input Congestion*). Input congestion occurs whenever the increasing one or more inputs decreases some outputs without improving other inputs or outputs. Conversely, congestion occurs when decreasing some of the inputs increases some outputs without worsening other inputs or outputs.

To clarify its relationship with technical inefficiency, we provide the definition of technical inefficiency which is as follows.

Definition 3 (*Technical Inefficiency*). Technical inefficiency is present when it is possible to improve some inputs or outputs without worsening other inputs or outputs.

An easy way to relate these definitions to each other is to regard technical inefficiency as synonymous with "waste". Thus, unlike the situation for technical efficiency, in the presence of technical inefficiency improvements may be effected without requiring further utilization of resources, or benefits in the form of reductions in outputs. This follows from the fact that improvements of inefficient inputs or outputs may be made without worsening other inputs or outputs. For congestion, moreover, a reduction in the congesting inputs is accompanied by improvement in one or more outputs without worsening other inputs or outputs.

Following Jahanshahloo and Khodabakhshi [12], we determine input congestion, by the input relaxation model, in two stages as follows, see also [6,16,17]. Using optimal solution $(\phi_o^*, \lambda^*, S_1^{-*}, S_2^{+*}, S^{+*})$ of the model (1), we solve the following model for determining technical inefficiency of inputs:

$$
\begin{aligned}
\text{Maximize} \quad & \sum_{i=1}^{m} \delta_i^{+} \\
\text{Subject to} \quad & (x_{io} - s_{i1}^{-*} + s_{i2}^{+*}) = \sum_{j=1}^{n} \lambda_j x_{ij} - \delta_i^{+}, \quad i = 1, \dots, m \\
& \phi_o^* y_{ro} + s_r^{+*} = \sum_{j=1}^{n} \lambda_j y_{rj}, \quad r = 1, \dots, s \\
& 1 = \sum_{j=1}^{n} \lambda_j \\
& \delta_i^{+} \le s_{i1}^{-*}, \quad i = 1, \dots, m \\
& \delta_i^{+}, \lambda_j \ge 0.
\end{aligned}
\tag{2}
$$

Finally, we define the amount of congestion for ith input as follows:

$$
s_i^{-c} = s_{i1}^{-*} - \delta_i^{+*}, \quad i = 1, \dots, m \tag{3}
$$

where δ_i^{+*} is obtained by solving the model (2). s_{i1}^{-c} is the amount of congesting input that causes producing less output, while δ_i^{+*} is maximum amount of technical inefficiency which is not congesting. Thus, if the amount of total slacks (total inefficiencies), s_{i1}^{-*}, and non-congesting input are the same, then we do not have any input congestion. In other words, in this case, the amount of input congestion is zero. While, as it is clear from (3), if the amount of total slack is more than non-congesting input, then the input congestion will exist for ith input. It is obvious that if s_{i2}^{+*} is positive for ith input, then there is no congestion for this input.

In the next section, we provide a one-model approach to estimate input congestion by the input relaxation model.

3.1. A one-model approach

We can replace the two previous models for determining input congestion by a single model, see [15] which provides an alternative procedure to determine input congestion by output oriented BCC model. Let $(\phi_o^*, \lambda^*, S_1^{-*}, S_2^{+*}, S^{+*})$ be an optimal solution for improving output model in (1). Using $s_i^{-c} = s_{i1}^{-*} - \delta_i^{+*}$ in (3) we can rewrite (2) as follows:

$$\begin{aligned}
\text{Maximize} \quad & \sum_{i=1}^{m} -s_i^{-c} \\
\text{Subject to} \quad & (x_{io} - s_i^{-c} + s_{i2}^{+*}) = \sum_{j=1}^{n} \lambda_j x_{ij}, \quad i = 1, \ldots, m \\
& \phi_o^* y_{ro} + s_r^{+*} = \sum_{j=1}^{n} \lambda_j y_{rj}, \quad r = 1, \ldots, s \\
& 1 = \sum_{j=1}^{n} \lambda_j \\
& 0 \leq s_i^{-c}, \quad i = 1, \ldots, m \\
& 0 \leq \lambda_j.
\end{aligned} \tag{4}$$

If we consider the following model:

$$\begin{aligned}
\text{Maximize} \quad & \phi_o + \varepsilon \left(\sum_{i=1}^{m} -s_i^{-c} + \sum_{r=1}^{s} s_r^{+} - \sum_{i=1}^{m} s_{i2}^{+} \right) \\
\text{Subject to} \quad & x_{io} = \sum_{j=1}^{n} \lambda_j x_{ij} + s_i^{-c} - s_{i2}^{+}, \quad i = 1, \ldots, m \\
& 0 = \sum_{j=1}^{n} \lambda_j y_{rj} - \phi_o y_{ro} - s_r^{+}, \quad r = 1, \ldots, s \\
& 1 = \sum_{j=1}^{n} \lambda_j \\
& s_i^{-c}, s_{i2}^{+}, \lambda_j, s_r^{+} \geq 0.
\end{aligned} \tag{5}$$

It is obvious that if $(\phi_o^*, \lambda^*, S_1^{-c*}, S_2^{+*}, S^{+*})$ is an optimal solution of (5), ϕ_o^*, S_2^{+*} and S^{+*} are part of an optimal solution of (1), and (λ^*, S_1^{-c*}) is an optimal solution of (4). In other words, we can regard model (4) as a part of a two-stage procedure for solving model (5). It is noticeable that the way in which we could normally solve problem (1) would be via a two-stage approach. That is, at the first stage, we maximize ϕ_o, and at the second stage, we maximize the sum of the slacks. Therefore, in addition to model (2), which we must solve to determine the amount of technical inefficiencies in inputs, it is necessary to solve two other problems. As a result, three problems are needed to be solved by the two-model approach for determining input congestion, while with the one-model approach, even if two-stage method is used to solve model (5), we just need to solve two problems. In fact, we reduced solving three problems, with two-model approach, to two problems with one-model approach. This is certainly important from computational point of view. We can state the following theorem in which s_i^{-c} represents the congesting amount of ith input.

Theorem 1. *Congestion is present if and only if in optimal solution $(\phi_o^*, \lambda^*, S_1^{-c*}, S_2^{+*}, S^{+*})$ of (5), at least one of the following conditions is satisfied:*

(i) $\phi_o^* > 1$ *and there exists at least one i* $(1 \leq i \leq m)$ *such that* $s_i^{-c*} > 0$.
(ii) *There exists at least one r* $(1 \leq r \leq s)$ *for which* $s_r^{+*} > 0$ *and also one i* $(1 \leq i \leq m)$ *such that* $s_i^{-c*} > 0$.

It is obvious that in both cases DMU_o is inefficient. In other words, if DMU_o has input congestion, then it is inefficient while the reverse is not true.

Theorem 2. *Congestion is present if and only if for an optimal solution $(\phi_o^*, \lambda^*, S_1^{-c*}, S_2^{+*}, S^{+*})$ of (5), there is at least one $s_i^{-c*} > 0$ $(1 \leq i \leq m)$.*

Proof. Necessary condition is obvious by the congestion definition. To prove sufficient condition, we prove that if $s_i^{-c*} > 0$, $\phi_o^* > 1$ or there exists at least one r $(1 \leq r \leq s)$ for which $s_r^{+*} > 0$. Suppose to the contrary that $\phi_o^* = 1$ and $s_r^{+*} = 0$ for all $r = 1, \ldots, s$. Let $\bar{\phi}_o = \phi_o^* = 1, \bar{s}_r = s_r^{+*} = 0, r = 1, \ldots, s, \bar{s}_i^{-c} = 0, i = 1, \ldots, m,\ \bar{s}_{i2}^{+} = 0, \bar{\lambda}_j = 0\ (j \neq 0)\ \&\ \bar{\lambda}_o = 1$. Then $(\bar{\phi}_o, \bar{\lambda}, \bar{S}_1^{-c}, \bar{S}_2^{+}, \bar{S}^{+})$ is a feasible solution of (5) for which

$$\begin{aligned}
\bar{\phi}_o + \varepsilon \left(\sum_{i=1}^{m} -\bar{s}_i^{-c} + \sum_{r=1}^{s} \bar{s}_r^{+} - \sum_{i=1}^{m} \bar{s}_{i2}^{+} \right) &= \phi_o^* > \phi_o^* + \left(\sum_{i=1}^{m} -s_i^{-c*} - \sum_{i=1}^{m} s_{i2}^{+*} \right) \\
&= \phi_o^* + \varepsilon \left(\sum_{i=1}^{m} -s_i^{-c*} + \sum_{r=1}^{s} s_r^{+*} - \sum_{i=1}^{m} s_{i2}^{+*} \right).
\end{aligned}$$

This is in contrast with the assumption that $(\phi_o^*, \lambda^*, S_1^{-c*}, S_2^{+*}, S^{+*})$ is an optimal solution of (5). $\square$

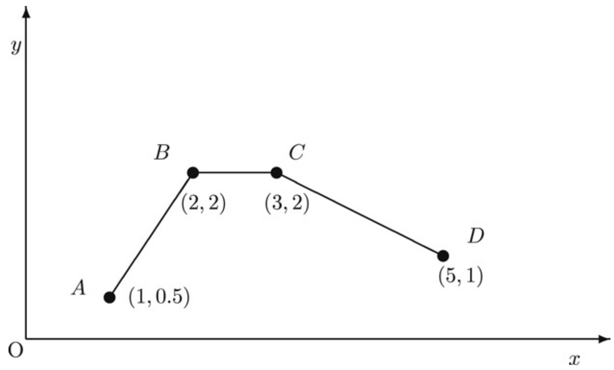

Fig. 1. Congestion.

Table 2
Determining congestion by the two-model approach, the models (1) and (2).

DMU	δ_i^{+*}	$s_i^{-c*} = s_{i1}^{-*} - \delta_i^{+*}$
A	0	0
B	0	0
C	1	0
D	1	2

Table 3
Determining congestion by the one-model approach, the model (5).

DMU	ϕ_o^*	s_i^{-c*}	s_{i2}^{+*}	s_r^{+*}
A	4	0	0	0
B	1	0	0	0
C	1	0	0	0
D	2	2	0	0

3.2. Illustrative example

We illustrate the method by recapturing data of Table 1 which are depicted in Fig. 1. The numerical results of the model (2) are presented in Table 2, while the results of the model (5) are presented in Table 3. The model (5) for DMU D of Fig. 1 becomes:

$$\begin{aligned}
\text{maximize} \quad & \phi_D + \varepsilon(-s^{-c} + s^{+} - s_{12}^{+}) \\
\text{subject to} \quad & 5 = \lambda_A + 2\lambda_B + 3\lambda_C + 5\lambda_D + s^{-c} - s_{12}^{+} \\
& 0 = 0.5\lambda_A + 2\lambda_B + 2\lambda_C + \lambda_D - 1\phi_D - s^{+} \\
& 1 = \lambda_A + \lambda_B + \lambda_C + \lambda_D \\
& s^{-c}, s^{+}, s_{12}^{+}, \lambda_A, \lambda_B, \lambda_C, \lambda_D \geq 0.
\end{aligned}$$

The optimal solution for DMU D is $\phi_D^* = 2, s^{+*} = 0, s^{-c*} = 2, \lambda_B^* = 1$, and other variables are zero. This solution shows that the DMU D has 2 units of congestion in its input, because s^{-c*} is equal to 2. In fact, a reduction from 5 units to 3 units in the input used by D could bring it into coincidence with C. This provides evidence of congestion in which a decrease of two units of input is associated with a one-unit increase in output obtained by moving from D to C. It is obvious that we can still reduce a one-unit of the input of DMU C without worsening output. But input reduction in moving from C to B is not associated with an output increase. Therefore, DMU C is just technically inefficient. In fact, one can improve the input of DMU C without worsening of its output based on the definition of technical inefficiency.

4. Empirical example

Now, we apply the model by using the input and outputs of 12 Canadian business schools which are presented in Table 4. The data was obtained from the website that Erkut [18] set up for his study on Canadian business schools.

Input:

Faculty members: All full-time continuing faculty members employed by a given business school in the academic year 2000–2001 (those with joint appointments were allocated to their primary employer.)

Outputs:

Total citation credits: the citation credits were based on the papers between 1990–1999 by the faculty members employed in 2000–2001 in each business school and adjusted for the number of co-authors.

Table 4
Data for 12 Canadian business schools.

DMU	University	Uprof	Citation	Paper
1	Carleton U.	30	227.85	41.95
2	McGill U.	66	771.13	112.26
3	McMaster U.	51	897.37	201.05
4	Queen's U.	56	660.87	90.52
5	Simon Fraser U.	47	251.4	78.03
6	U. of Alberta	71	682.53	133.52
7	U. of British Columbia	76	1860.12	286.78
8	U. of New Brunswick at Fredericton	32	158.83	64.83
9	U. of Toronto	85	1341.55	208.69
10	U. of Victoria	23	98.5	32.92
11	U. of Waterloo	38	207.83	63.33
12	U. of Western Ontario	70	978.02	88.05

Table 5
Computational results by model (5), one-model approach.

DMU	University	ϕ^*	s_i^{-c*}	s_{12}^{+*}	s_1^{+*}	s_2^{+*}
1	Carleton U.	6.836234	0	46	302.48	0
2	McGill U.	2.4122	0	10	0	15.99
3	McMaster U.	1.426411	0	25	580.1	0
4	Queen's U.	2.814653	0	20	0	32
5	Simon Fraser U.	3.675253	0	29	936.16	0
6	U. of Alberta	2.147843	0	5	394.15	0
7	U. of British Columbia	1	0	0	0	0
8	U. of New Brunswick at Fredericton	4.423569	0	44	1157.52	0
9	U. of Toronto	1.374191	9	0	16.57	0
10	U. of Victoria	8.711422	0	53	1002.04	0
11	U. of Waterloo	4.528344	0	38	918.99	0
12	U. of Western Ontario	1.901924	0	6	0	119.32

Total paper credits: the number of papers between 1990–1999 by the faculty members employed in 2000–2001 in each business school and adjusted for the number of co-authors.

The data in Erkut [18] was extracted from the database of the Institute of Scientific Information (ISI). The study of the papers was based on a 10-year time window (1990–1999) and the citation study is based on an 11.4-year time window (1990–2001, May).

The computational results of the model (5), one-model approach, for determining input congestion by the output improvement model, are shown in Table 5. The first and second columns of the Table show DMU number and university name, respectively. Column 3 represents the value of the ϕ^*, and columns 4 and 5 show the values of input congestion and increment amount of the university professors, in sequence. Finally, the last two columns show the slacks for citation and paper counts. Based on the results of Table 5, just DMU 7, U. of British Columbia, is DEA efficient. In fact, ϕ^* of this DMU is equal to 1, and all its slacks including s^{-c*}, value of input congestion, are zero. $\phi^* > 1$ indicates DMU_o could increase its output proportionally, ϕ^* times of its current output, to be DEA efficient. For example, DMU 2, McGill university, has $\phi^* = 2.41, s_{12}^{+*} = 10, s_2^{+*} = 15.99$ which means if this DMU used 10 extra university professors, it could produce 1858.42 $(= 2.41 * 771.13)$ citations and 286.54 $(= 2.41 * 112.26 + 15.99)$ papers to be efficient. For the rest of the DMUs, the lower the value of ϕ^* the better the DMU. Top six schools are DMU 7, U. of British Columbia, 1, DMU 9, Toronto, 1.37, DMU 3, McMaster, 1.42, DMU 12, UWO, 1.90, DMU 6, U. of Alberta, 2.14 and DMU 2, McGill university, 2.41. Although DMU 9, U. of Toronto, is the second school which is most productive, it has 9 congesting university professors. Therefore, if U. of Toronto reduced its input as much as 9 university professors, it could produce $1.37 * (1341.55, 208.69) + (16.75, 0)$ outputs. It is noticeable that the rest of DMUs have no congesting inputs (see Table 2). The last two columns of Table 2 show that slack of citation grows more dramatically than that of the paper count. It is quite meaningful, because citation counts, usually, grows more than the paper counts.

Computational results of the two-model approach, also, are presented in Tables 6 and 7. Comparing numerical results of the one-model approach and the two-model approach shows that the value of input congestion are identical by the two approaches. However, the one-model approach is preferred from computational point of view.

5. Conclusion

In this paper, we provided a one-model approach for determining input congestion in data envelopment analysis based on improved outputs. As an empirical example, data of 12 Canadian business schools were used to apply the model. Note that removal of non-productive tenure-stream faculty members will eliminate the congesting component in the input slack. Erkut [18] pointed out: "If the age distributions of the business schools are similar among the business schools, the aggregate statistics produced will be meaningful". We do recognize, however, that schools which have experienced a large number

Table 6
Computational results by model (1).

DMU	University	ϕ^*	s_{11}^{-*}	s_{12}^{+*}	s_1^{+*}	s_2^{+*}
1	Carleton U.	6.836234	0	46	302.48	0
2	McGill U.	2.4122	0	10	0	15.99
3	McMaster U.	1.426411	0	25	580.1	0
4	Queen's U.	2.814653	0	20	0	32
5	Simon Fraser U.	3.675253	0	29	936.16	0
6	U. of Alberta	2.147843	0	5	394.15	0
7	U. of British Columbia	1	0	0	0	0
8	U. of New Brunswick at Fredericton	4.423569	0	44	1157.52	0
9	U. of Toronto	1.374191	9	0	16.57	0
10	U. of Victoria	8.711422	0	53	1002.04	0
11	U. of Waterloo	4.528344	0	38	918.99	0
12	U. of Western Ontario	1.901924	0	6	0	119.32

Table 7
Determining congestion by the two-model approach, the models (1) and (2).

DMU	s_{11}^{-*}	δ_1^{+*}	$s_1^{-c*} = s_{11}^{-*} - \delta_1^{+*}$
9	9	0	9
Other DMUs	0	0	0

of recent retirements (and recent hires) may be negative impacted in this study. Thus to have a better understanding as to why the congestions exist, it would be very interesting to examine the distributions of the age and the ranks of the tenure stream professors among the business schools in our study. We would say that the business schools with congestions might have many ready-to-retire professors who have not published for quite a long time. We also want to point out that although U. of Toronto (U. of T) is listed among the top nine most productive business schools in [18], our study showed that it has serious congestion problems with 9 input congestion. It is very interesting to note that these issues were not discussed in [18] because the approaches used in their study are different. From computational point of view, proposed one-model approach is easier to use than the two-model approach introduced in [12]. Finally, study of the incorporation of the proposed deterministic model to its chance constrained form can be suggested for further research, see Khodabakhshi and Asgharian [19].

Acknowledgments

The author would like to thank an anonymous referee for his (her) suggestions on this paper. The author, also, would like to express his thanks to Vahid Omrani for editing the manuscript of the paper at hand.

References

[1] A. Charnes, W.W. Cooper, E. Rhodes, Measuring the efficiency of DMUs, European Journal of Operational Research 2 (6) (1978) 429–444.
[2] R.D. Banker, A. Charnes, W.W. Cooper, Some models for estimating technical and scale inefficiencies in data envelopment analysis, Management Science 30 (1984) 1078–1092.
[3] R. Färe, L. Svensson, Congestion of factors of production, Econometrica 48 (1980) 1743–1753.
[4] R. Färe, S. Groskopf, Measuring congestion in production, Zeitschrift fur Nationalokonomie 43 (1983) 257–271.
[5] R. Färe, S. Groskopf, C.A.K. Lovell, The Measurement of Efficiency of Production, Kluwer Nijhoff Publishing, Boston, 1985.
[6] W.W. Cooper, R.G. Thompson, R.M. Thrall, Extensions and new developments in DEA, Annals of Operations Research 66 (1996) 3–45.
[7] L. Cherchye, T. Kuosmanen, T. Post, Alternative treatments of congestion in DEA: A rejoinder to Cooper, Gu and Li, European Journal of Operational Research 132 (2001) 75–80.
[8] W.W. Cooper, Bisheng Gu, Shanling Li, Comparisons and evaluations of alternative approaches to the treatment of congestion in DEA, European Journal of Operational Research 132 (2001) 62–74.
[9] W.W. Cooper, Bisheng Gu, Shanling Li, Note: Alternative treatments of congestion in DEA — A response to the cherchye, Kuosmanen and Post critique, European Journal of Operational Research 132 (2001) 81–87.
[10] W.W. Cooper, L.M. Seiford, J. Zhu, Slacks and congestion: Response to a comment by R. Färe and S. Grosskopf, Socio-Economic Planning Sciences 35 (2001) 205–215.
[11] W.W. Cooper, L.M. Seiford, J. Zhu, A unified additive model approach for evaluating inefficiency and congestion with associated measures in DEA, Socio-Economic Planning Sciences 34 (2000) 1–25.
[12] G.R. Jahanshahloo, M. Khodabakhshi, Suitable combination of inputs for improving outputs in DEA with determining input congestion – Considering textile industry of China –, Applied Mathematics and Computation 151 (1) (2004) 263–273.
[13] G.R. Jahanshahloo, M. Khodabakhshi, Determining assurance interval for non-Archimedean element in the improving outputs model in DEA, Applied Mathematics and Computation 151 (2) (2004) 501–506.
[14] M. Khodabakhshi, A super-efficiency model based on improved outputs in data envelopment analysis, Applied Mathematics and Computation 184 (2) (2007) 695–703.
[15] W.W. Cooper, Honghui Deng, Zhimin M. Huang, Susan X. Li, A one-model approach to congestion in data envelopment analysis, Socio-Economic Planning Sciences 36 (2002) 231–238.
[16] R. Färe, S. Groosskoff, When can slacks be used to identify congestion? An aswer to W.W. Cooper, L. Seiford and J. Zhu, Socio-Economic Planning Sciences 35 (2001) 217–221.

[17] W.W. Cooper, Honghui Deng, Bisheng Gu, Shanling Li, R.M. Thrall, Using DEA to improve the management of congestion in chinese industries (1981–1997), Socio-Economic Planning Sciences 35 (2001) 227–242.
[18] E. Erkut, Measuring Canadian business school research output and impact, Canadian Journal of Administrative Sciences 19 (2002) 97–123.
[19] M. Khodabakhshi, M. Asgharian, An input relaxation measure of efficiency in stochastic data envelopment analysis, Applied Mathematical Modelling 33 (2009) 2010–2023.